Confluent Term Rewriting Systems

Yoshihito Toyama

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A Quarter Century Ago ..

Kokich Futatsugi and Yoshihito Toyama
Term rewriting systems and their applications: A survey
J. IPS Japan 24 (2) (1983) 133-146, *in Japanese*.


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解説

項書き換え型計算モデルとその応用†

二木厚吉†† 外山芳人†††

1. はじめに

2. 項書き換えシステムとは？

Contents of Survey (1983)

1. Introduction
2. What is term rewriting system
3. Theory of term rewriting systems

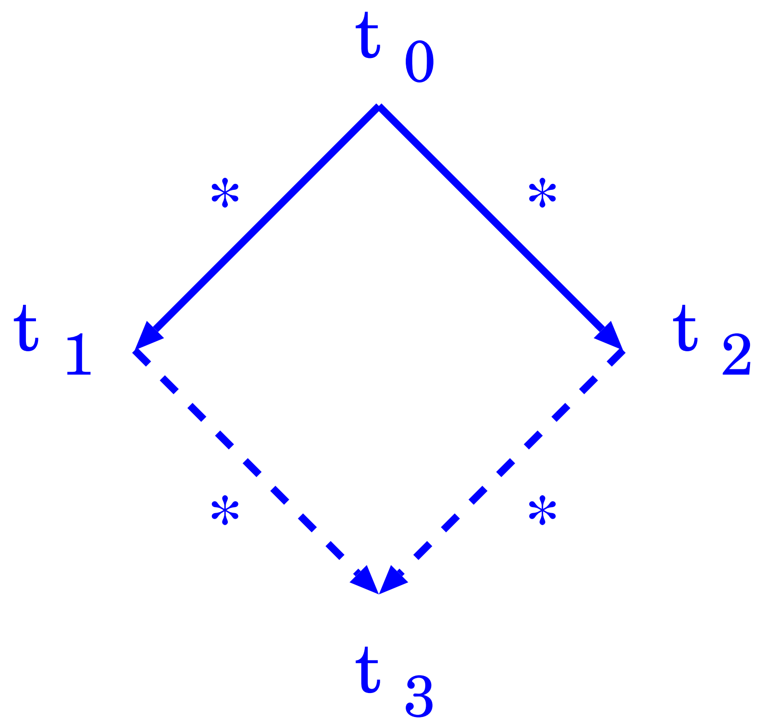
   confluence, termination, Knuth-Bendix completion,
   strategies (by Toyama)

4. Applications of term rewriting systems

   algebraic specification, program transformation,
   equational program (by Futatsugi)

5. Conclusion

Confluence



Confluence implies at most one normal form for any term. Thus, confluent term rewriting systems give flexible computation and effective deduction for equational systems.

Classical Criteria for Confluence

When the survey (1983) was planed, we knew only three confluence criteria:

- Terminating TRS is confluent iff all critical pairs in it are joinable (Knuth and Bendix 1970).
- Left-linear non-overlapping TRS is confluent (Rosen 1973).
- Left-linear parallel-closed TRS is confluent (Huet 1980).

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  TRS is terminating if every reduction terminates.

Confluence Criterion for Terminating TRS

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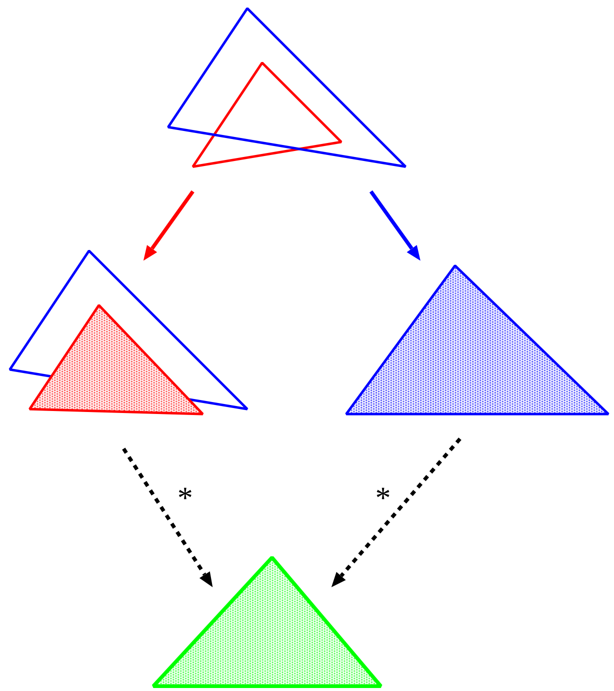

Thus confluence of terminating TRSs is decidable.

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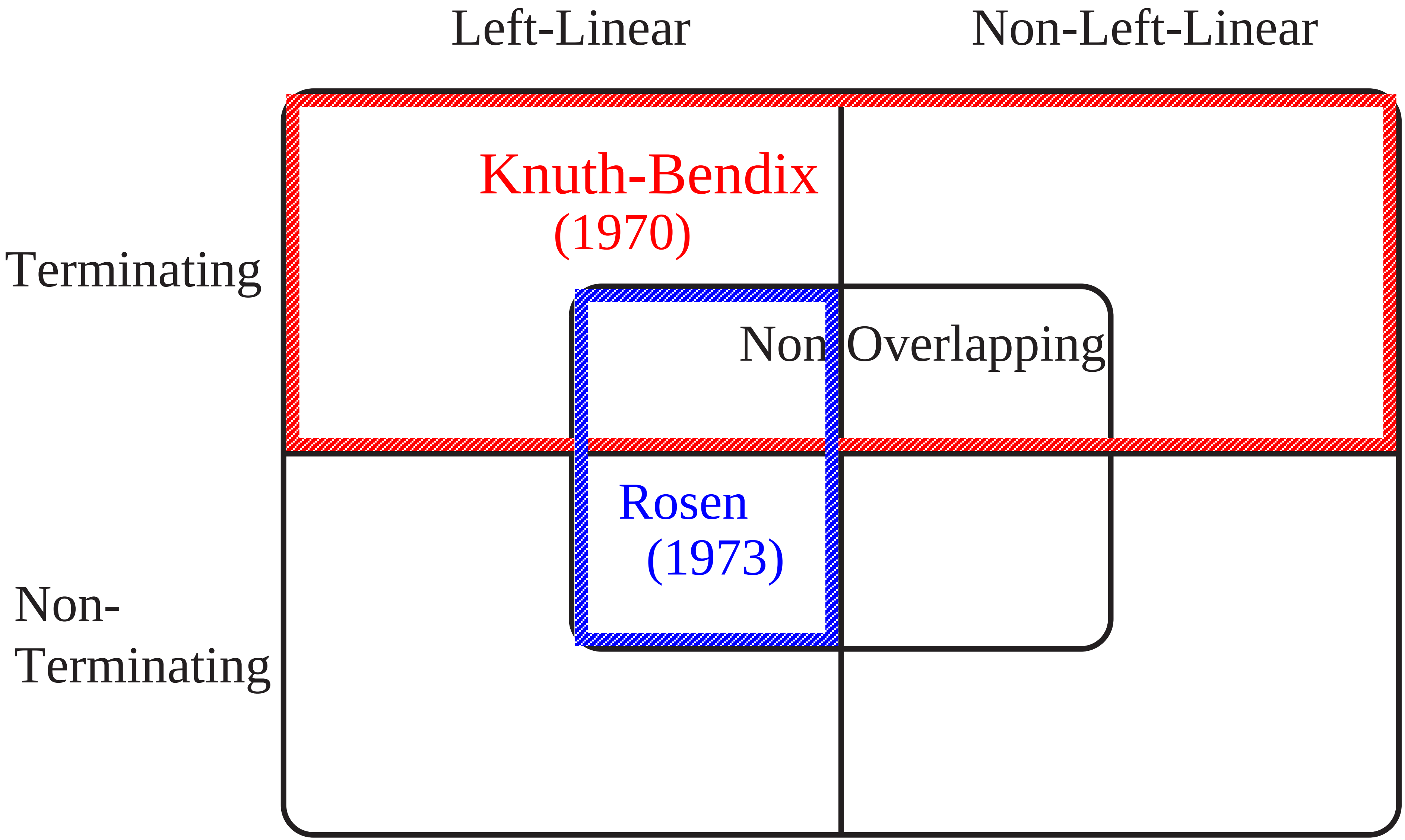
Term is linear if no variable occurs more than once.

TRS is left-linear if the left-hand side is linear for every rewrite rule.

TRS is non-overlapping if it has no critical pairs.

Confluence Criteria (30 years ago)



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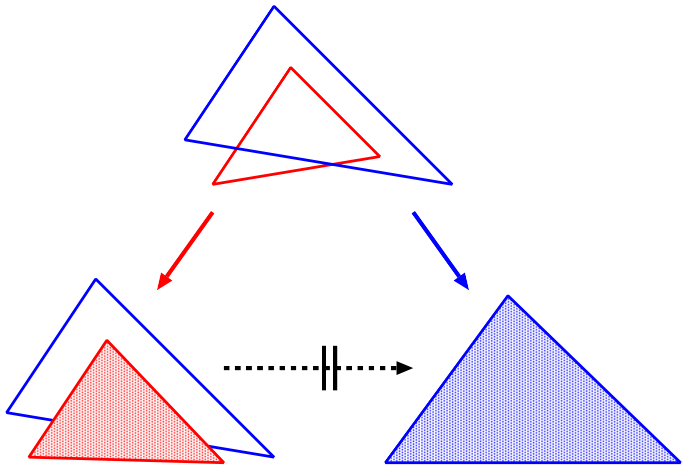
- Left-linear parallel-closed TRS is confluent (Huet 1980).

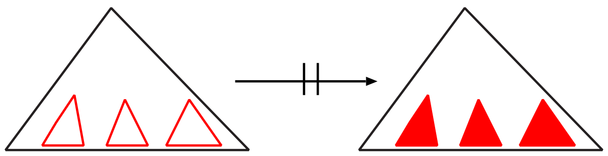
Huet criterion for left-linear TRS was extended by Toyama (1981, 1988), van Oostrom (1995), Gramlich (1996), Oyamaguchi and Ohta (1997, 2003), Okui (1998), et al.

Confluence Criterion (Huet 1980)

- **Left-linear TRS is confluent if every critical pair satisfies**



Parallel reduction is defined by



Confluence Criterion (Toyama 1988)

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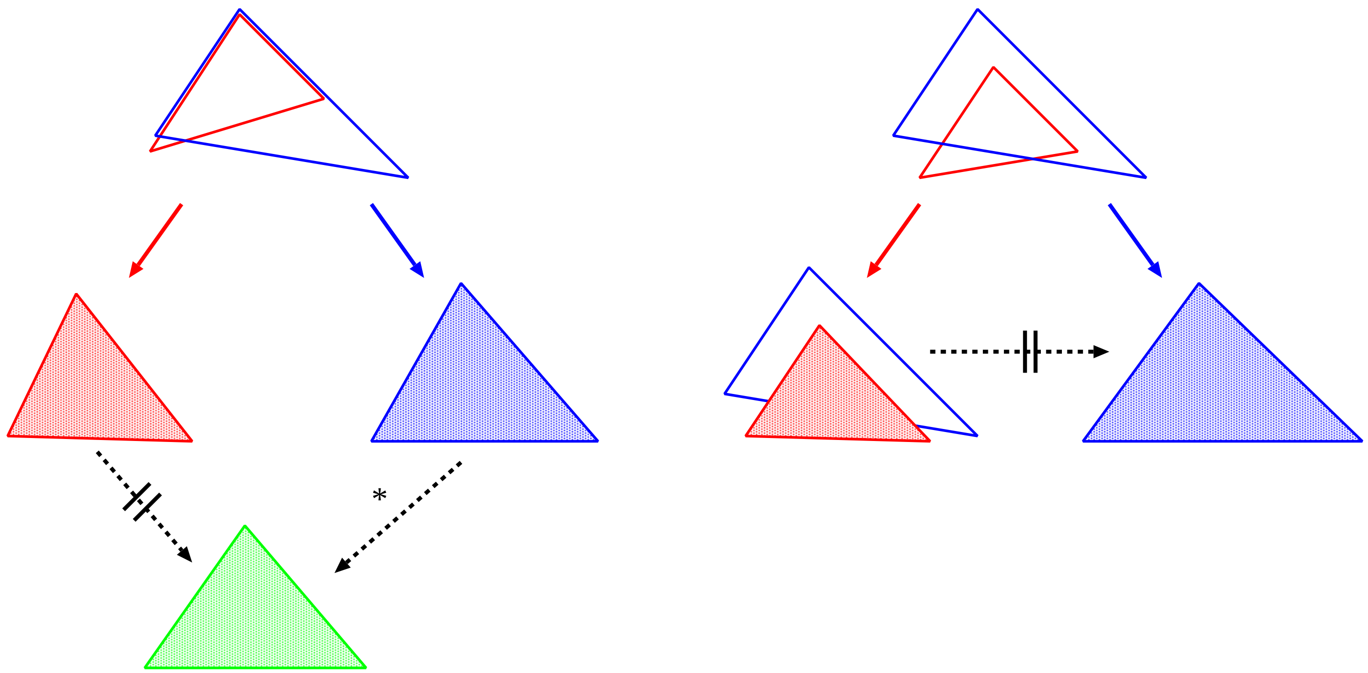

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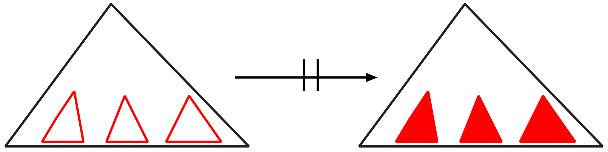
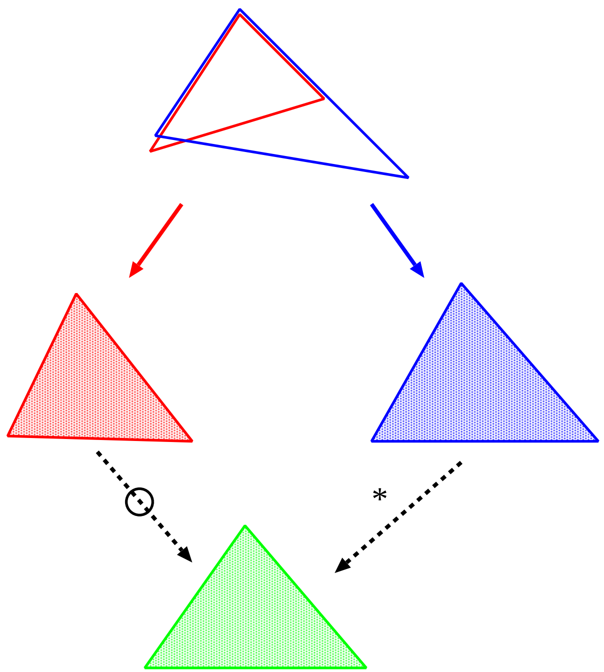
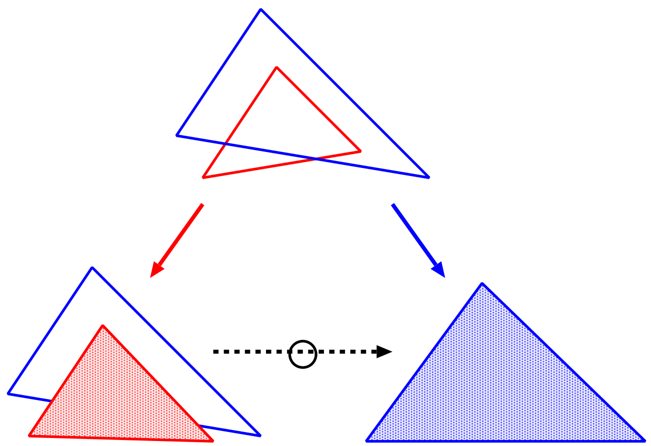


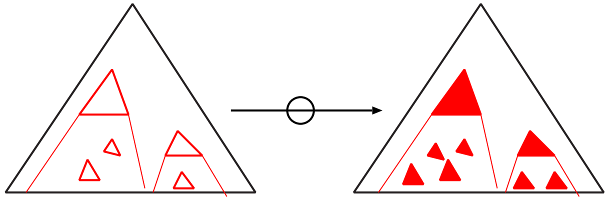
Confluence Criterion (van Oostrom 1995)

- Left-linear TRS is confluent if every critical pair satisfies

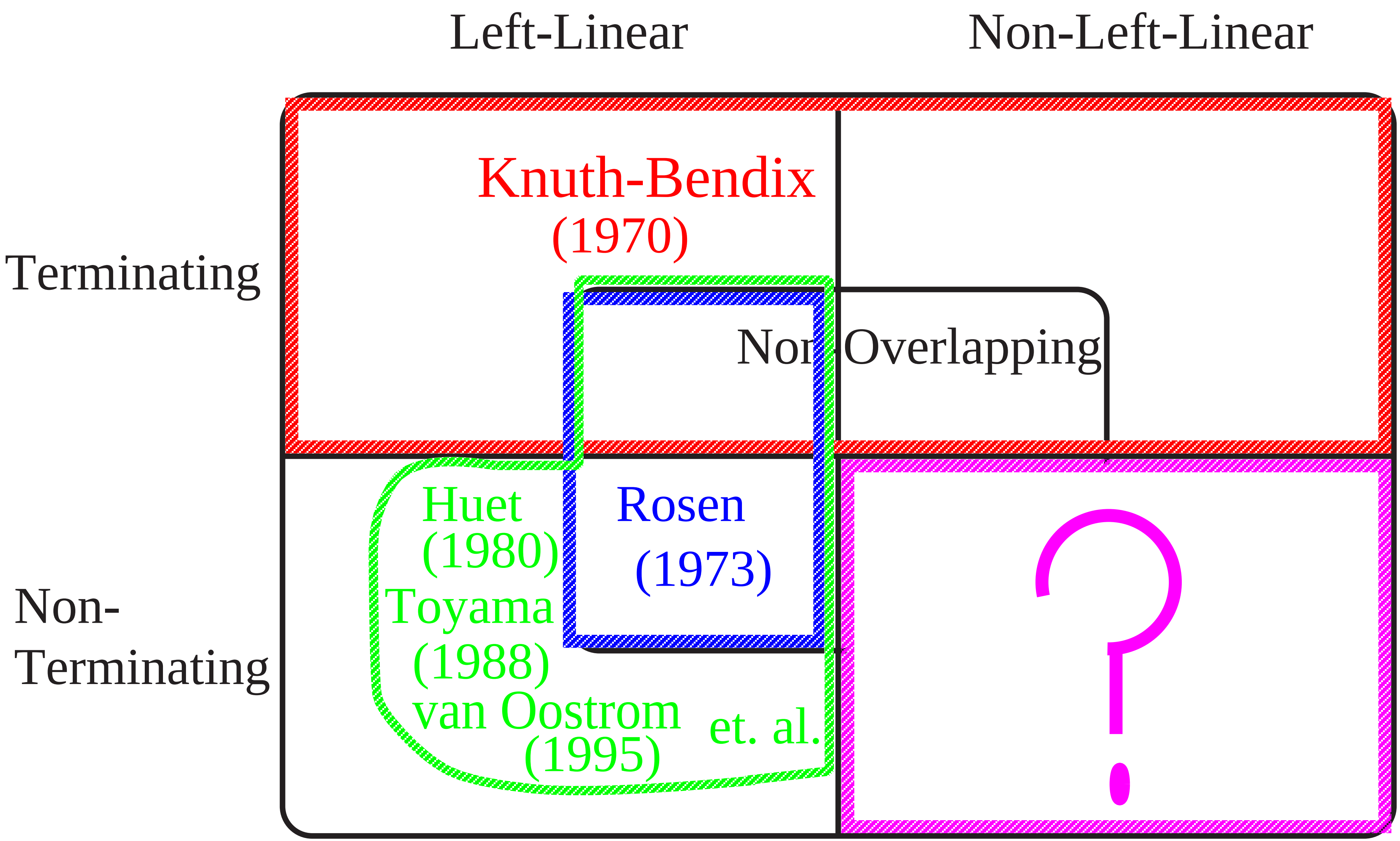




Development reduction is defined by



Confluence Criteria

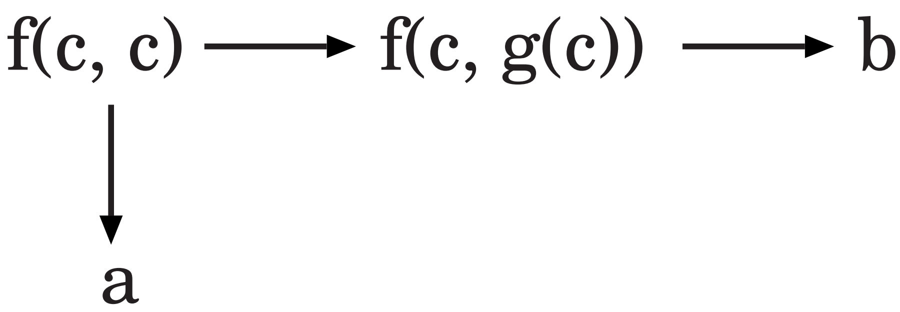


Criteria for Non-Left-Linear Non-Terminating TRS?

Non-overlapping does not imply confluence for non-left-linear non-terminating TRSs.

$$R \begin{cases} f(x,x) \longrightarrow a \\ f(x,g(x)) \longrightarrow b \\ c \longrightarrow g(c) \end{cases}$$

(Huet 1980)



Criteria for Non-Left-Linear Non-Terminating TRS?

Questions:

- Is a left-linear non-overlapping TRS + $\{D(x,x) \rightarrow E\}$ confluence? (Staples 1975)
- Is a left-linear non-overlapping TRS + parallel-if confluence?

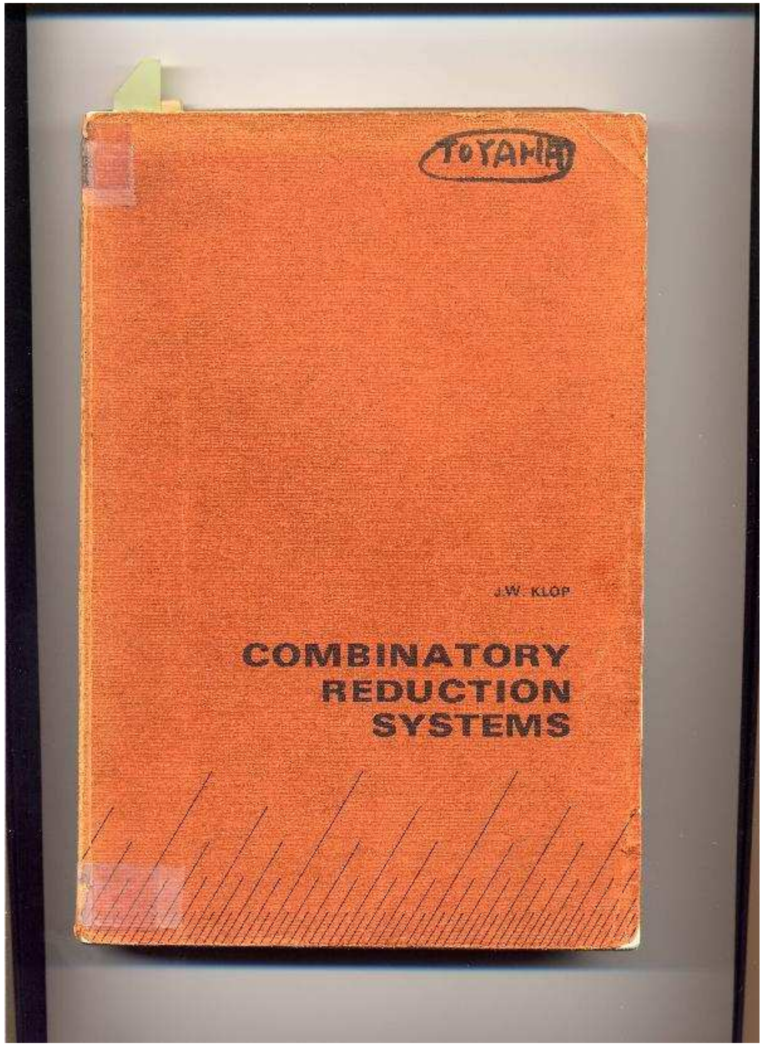
$$\text{parallel-if} \begin{cases} \mathsf{if}(\mathsf{true}, x, y) \longrightarrow x \\ \mathsf{if}(\mathsf{false}, x, y) \longrightarrow y \\ \mathsf{if}(z, x, x) \longrightarrow x \end{cases}$$

(O'Donnell 1977)

Note that we cannot apply all the confluence criteria which have been mentioned to this problem.

Combinatory Reduction Systems (Klop 1980)

Answers:



Negative Result by Klop

$CL + \{Dxx \to E\}$ **is not confluent.**

$$CL \begin{cases} Sxyz \to (xz)(yz) \\ Kxy \to y \end{cases}$$

Negative Result by Klop

$$R \begin{cases} A \to CA \\ Cz \to Dz(Cz) \\ Dzz \to E \end{cases}$$

A → CA → DA(CA) → D(CA)(CA) → E

CA
↓
C(CA)
↓ *
CE
↓
DE(CE)
↓
DE(DE(CE))
↓
⋮

Positive Result by Klop

$CL + \{Dxx \to E\}$ **is not confluent. But**

$CL + \{D(x, x) \to E\}$ **is confluent (Klop 1980).**

This is the first non-trivial example of confluent non-left-linear non-terminating TRS.

Question:

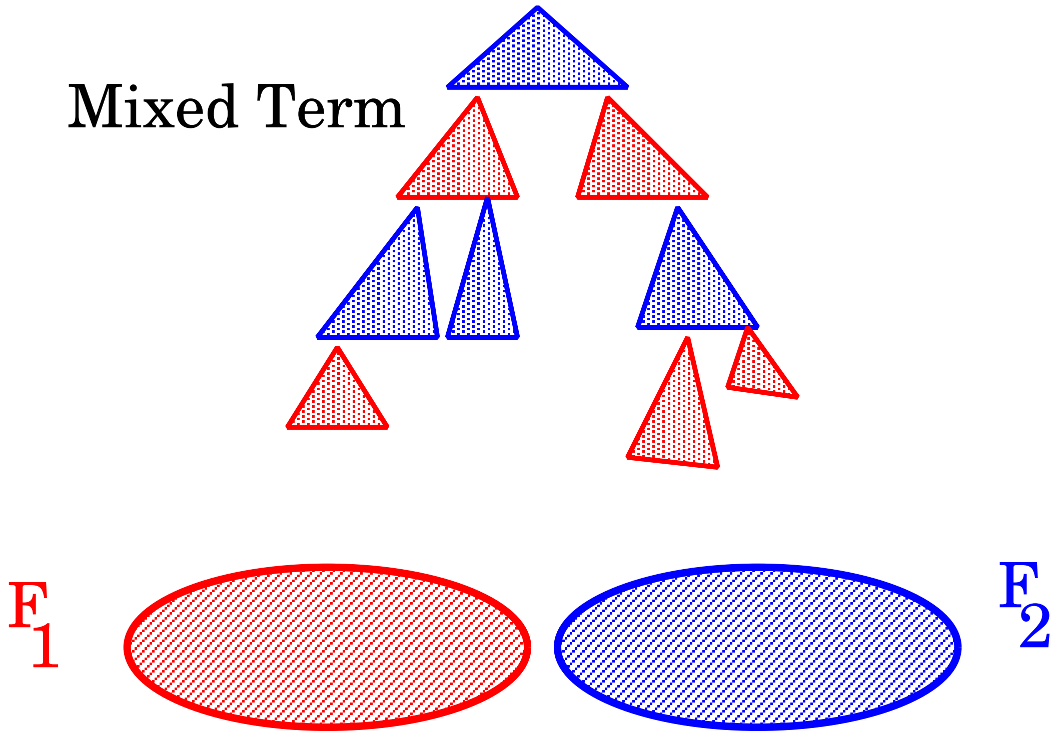
What is the essential difference between them?

Answer:

Modularity (Toyama 1987)

Direct Sum of TRSs

Let $\mathcal{R}_1$ on $\mathcal{F}_1$ and $\mathcal{R}_2$ on $\mathcal{F}_2$ be two TRSs with $\mathcal{F}_1 \cap \mathcal{F}_2 = \phi$. Then the direct sum $\mathcal{R}_1 \oplus \mathcal{R}_2$ is defined as the new TRS $\mathcal{R}_1 \cup \mathcal{R}_2$ on $\mathcal{F}_1 \cup \mathcal{F}_2$.



Modularity of Confluence (Toyama 1987)

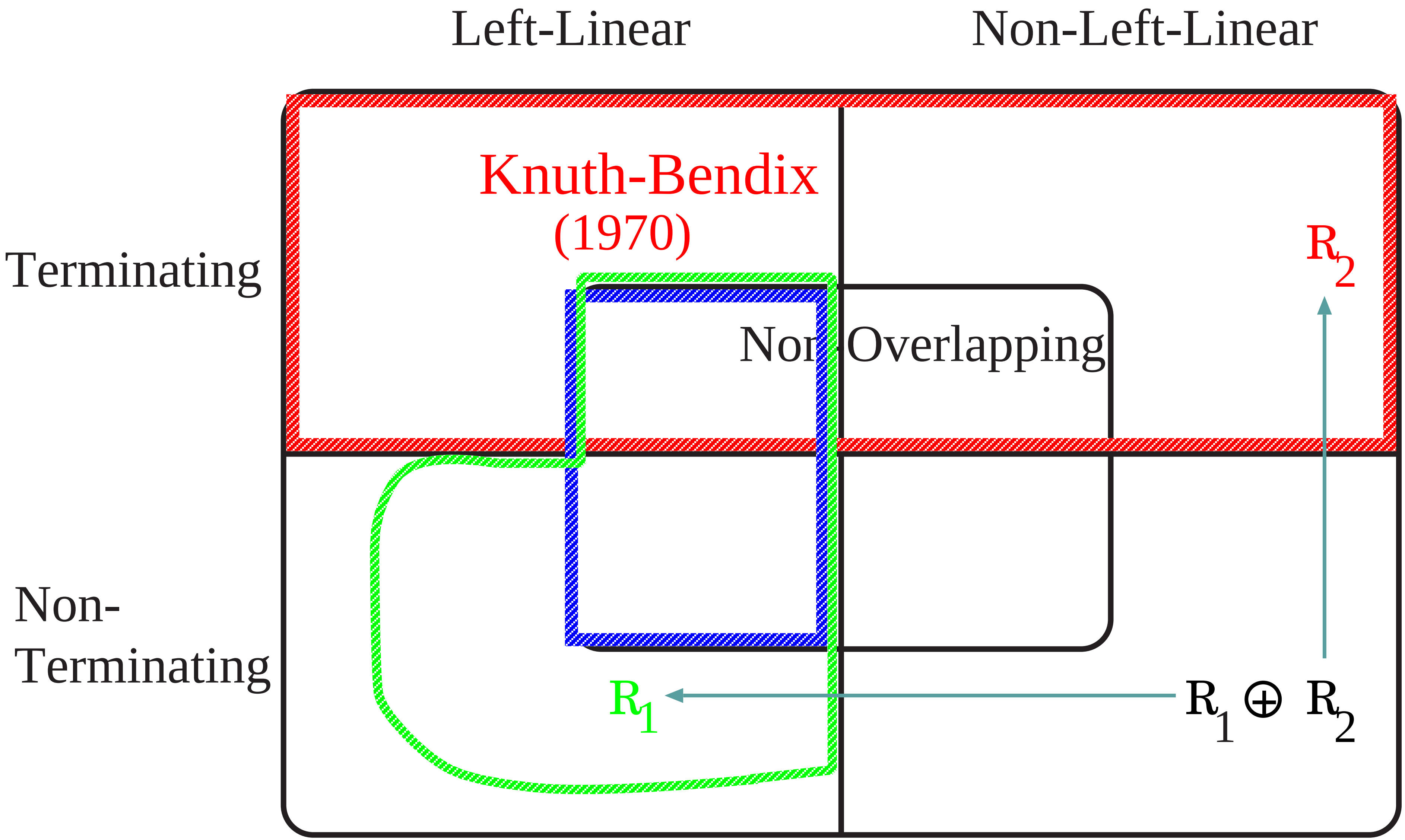
R_1 and R_2 are confluent $\Longleftrightarrow$ $R_1 \oplus R_2$ is confluent.

Example: Let $\mathcal{R}$ on $\mathcal{F}$ be a left-linear non-overlapping TRS, and let $\mathcal{F} \cap \{\mathsf{if}, \mathsf{true}, \mathsf{false}\} = \phi$. Then $\mathcal{R}$ + parallel-if is confluent.

$$\text{parallel-if} \begin{cases} \mathsf{if}(\mathsf{true}, x, y) \to x \\ \mathsf{if}(\mathsf{false}, x, y) \to y \\ \mathsf{if}(z, x, x) \to x \end{cases}$$

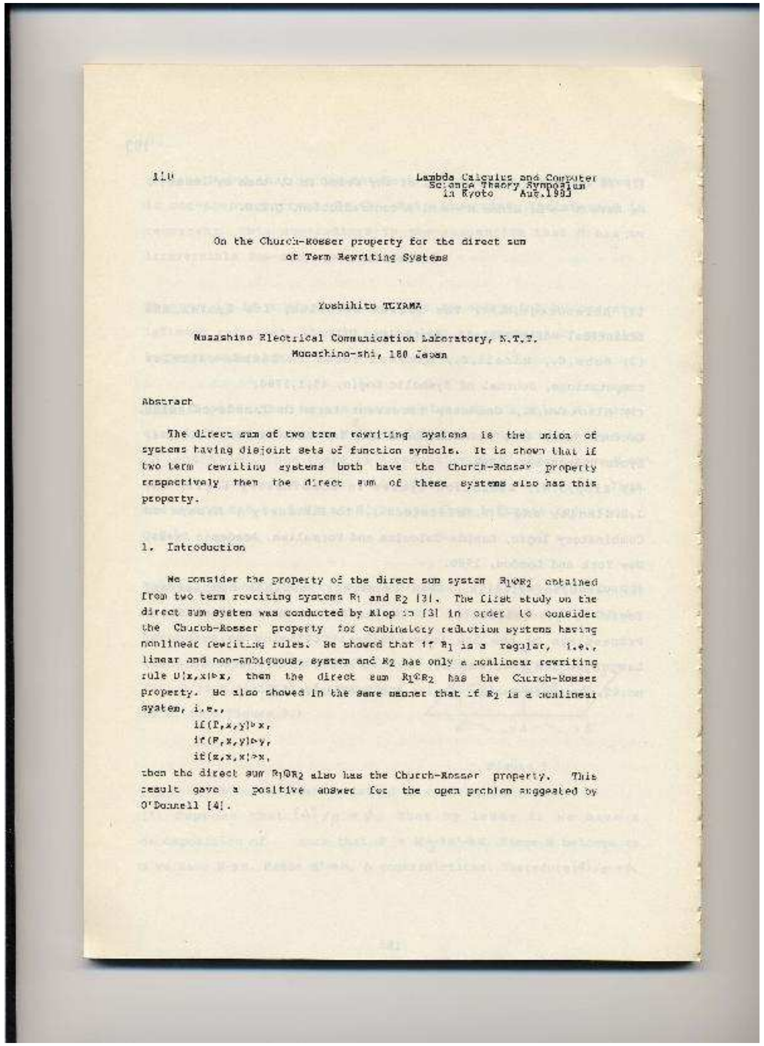
Note that $\mathcal{R}$ is confluent from Rosen criterion, and parallel-if is confluent from Knuth-Bendix criterion.

Modularity of Confluence



Modularity of Confluence

I presented my result in a small workshop at Kyoto, 1983.



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Lambda Calculus and Computer Science Theory Symposium in Kyoto Aug.1983

On the Church-Rosser property for the direct sum of Term Rewriting Systems

Yoshihito TOYAMA

Musashino Electrical Communication Laboratory, N.T.T. Musashino-shi, 180 Japan

Abstract

The direct sum of two term rewriting systems is the union of systems having disjoint sets of function symbols. It is shown that if two term rewriting systems both have the Church-Rosser property respectively then the direct sum of these systems also has this property.

1. Introduction

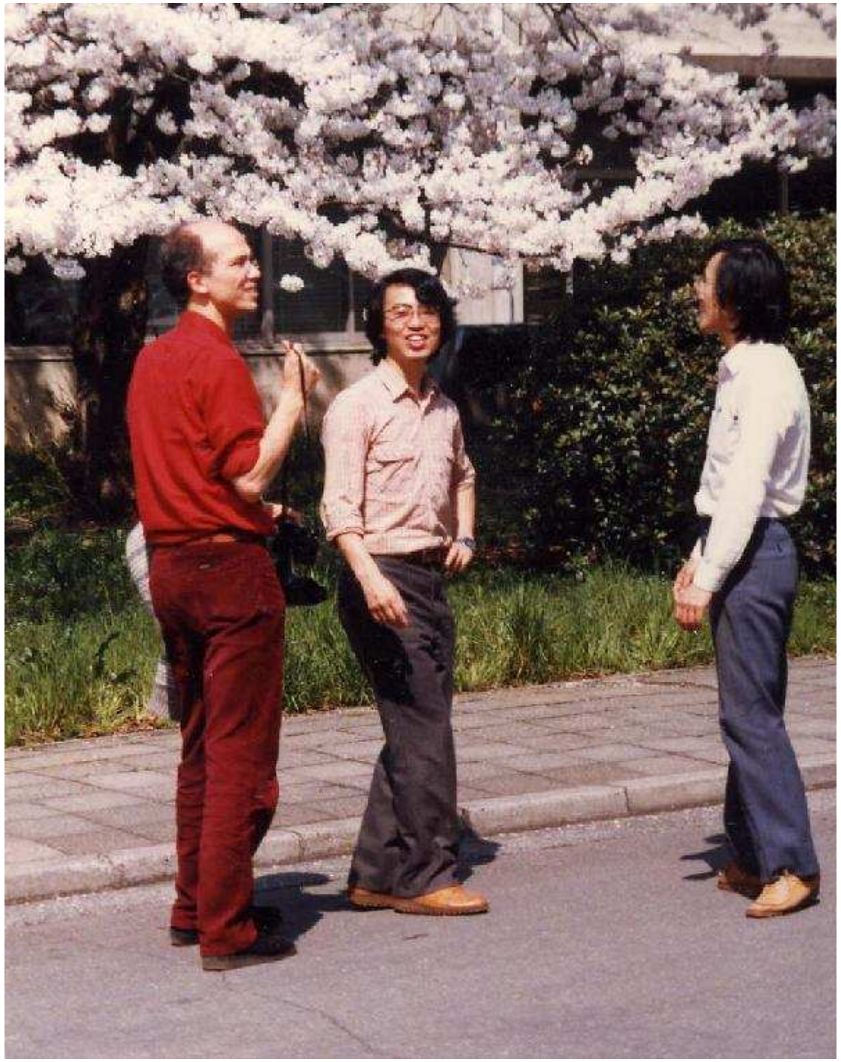
We consider the property of the direct sum system $R_1 \oplus R_2$ obtained from two term rewriting systems R_1 and R_2 [3]. The first study on the direct sum system was conducted by Klop in [3] in order to consider the Church-Rosser property for combinatory reduction systems having nonlinear rewriting rules. He showed that if R_1 is a regular, i.e., linear and non-ambiguous, system and R_2 has only a nonlinear rewriting rule $D(x,x) \triangleright x$, then the direct sum $R_1 \oplus R_2$ has the Church-Rosser property. He also showed in the same manner that if R_2 is a nonlinear system, i.e.,

$if(T,x,y) \triangleright x,$
$if(F,x,y) \triangleright y,$
$if(z,x,x) \triangleright x,$

then the direct sum $R_1 \oplus R_2$ also has the Church-Rosser property. This result gave a positive answer for the open problem suggested by O'Donnell [4].

Modularity of Confluence

Barendregt participated in the same workshop. He asked ...



Contradiction to Klop's Example?

$CL + \{Dxx \to E\}$ is not confluent (Klop 1980).

$$CL \begin{cases} Sxyz \to (xz)(yz) \\ Kxy \to y \end{cases}$$

CL and $\{Dxx \to E\}$ are confluent respectively,
and $\{S, K\} \cap \{D\} = \phi$.
From the modularity $CL + \{Dxx \to E\}$ should be confluent.
Does it contradict Klop's example?

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From the modularity $CL + \{Dxx \to E\}$ should be confluent.
Does it contradict Klop's example?

This is not the case.

Contradiction to Klop's Example?

$CL + \{(D \bullet x) \bullet x \to E\}$ is not confluent (Klop 1980).

$$CL \begin{cases} ((S \bullet x) \bullet y) \bullet z \to (x \bullet z) \bullet (y \bullet z) \\ (K \bullet x) \bullet y \to y \end{cases}$$

$CL + \{Dxx \to E\}$ is **not** direct sum since $\{S, K, \bullet\} \cap \{D, \bullet\} = \{\bullet\}$.

Modularity of Termination

I submitted my result to J. ACM and received the referee reports in 1984, in which one referee asked

"Can the author prove by his analysis of of the layer structure of $\mathcal{R}_1 \oplus \mathcal{R}_2$ - terms also the following:

R_1 and R_2 are terminating $\iff$ $R_1 \oplus R_2$ is terminating?

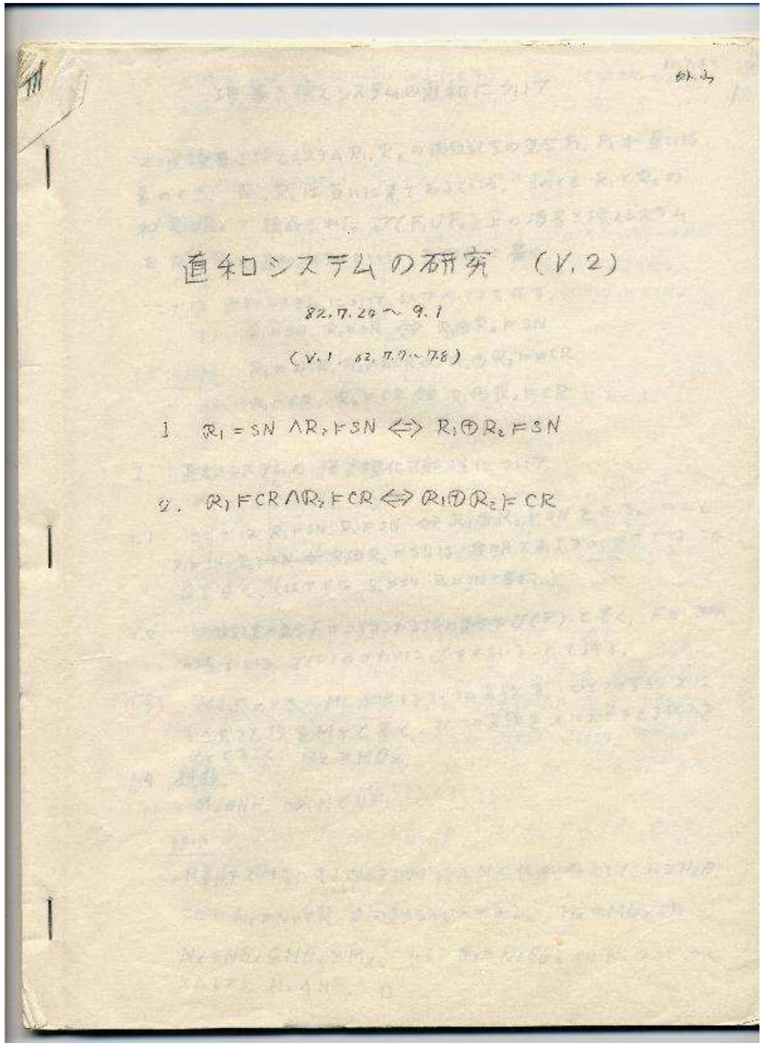
Maybe this fact, which would also be whorthwhile to have, can be obatained with relatively little extra effort."

My answer for the question was completely **YES**, because ...

Modularity of Termination

I had already proved the fact:

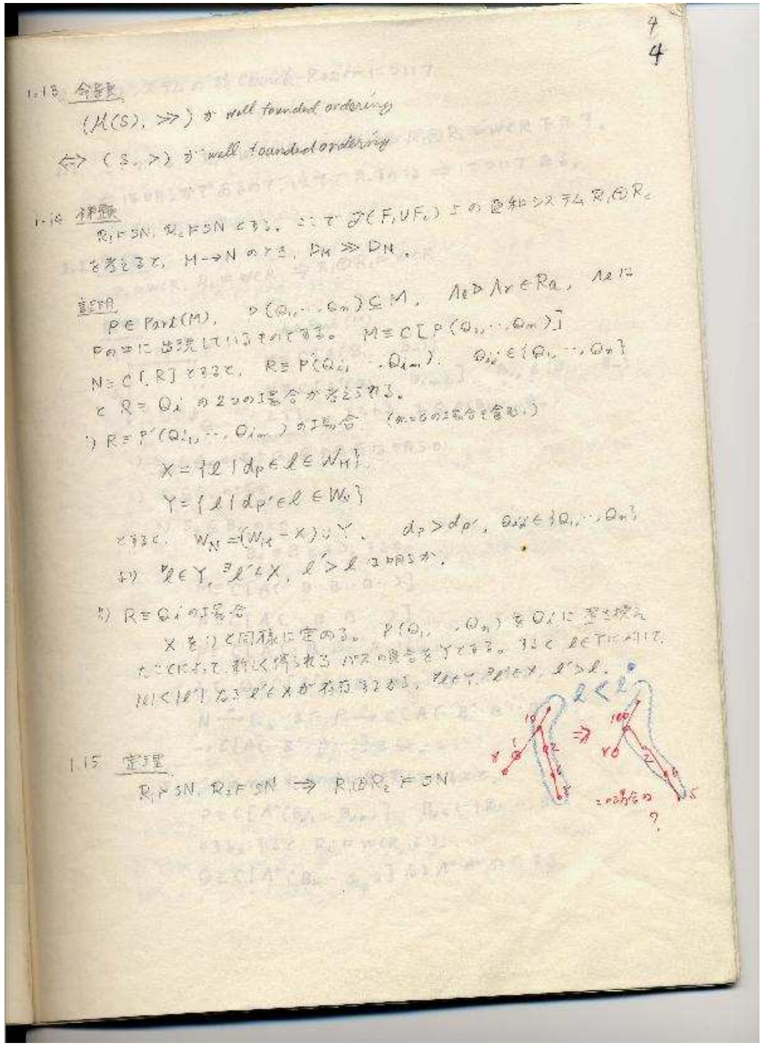
R_1 **and** R_2 **are terminating** $\Longleftrightarrow$ $R_1 \oplus R_2$ **is terminating.**



Modularity of Termination

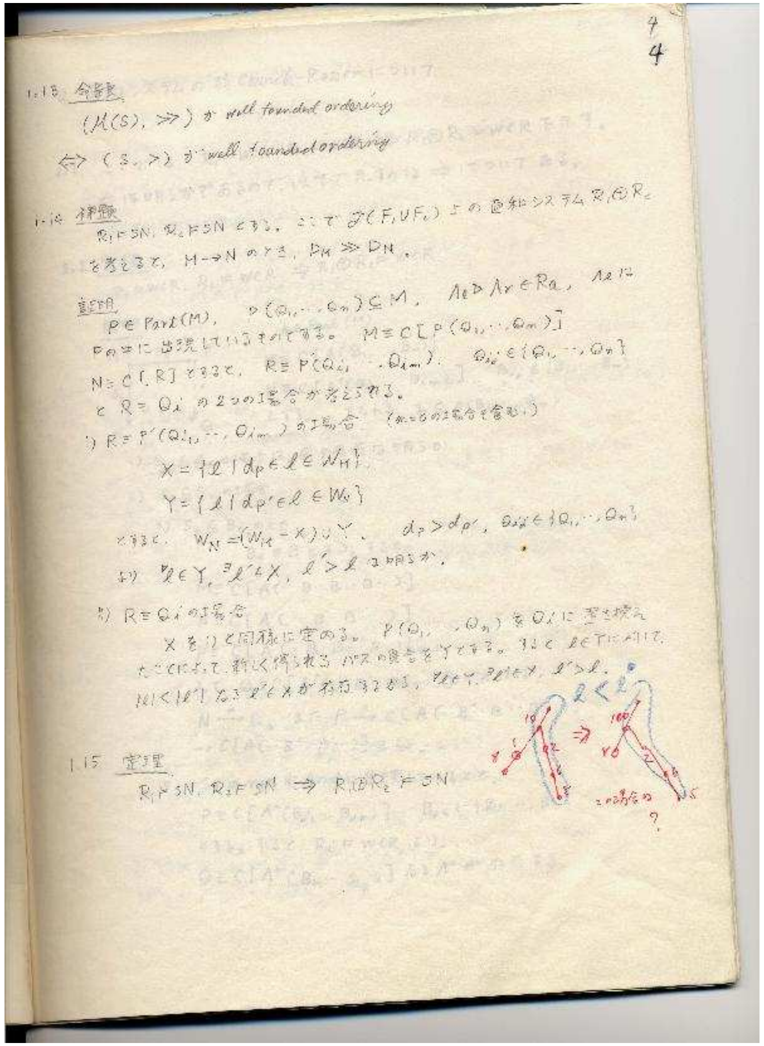
The following page concludes that:

R_1 **and** R_2 **are terminating** $\Longleftrightarrow$ $R_1 \oplus R_2$ **is terminating.**



Modularity of Termination

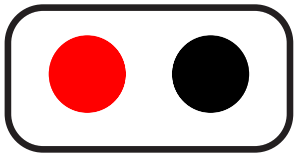
I tried to complete my proof by adding the details to the following sketch. But ...



Modularity of Termination

A proof of one assumption always produced another new assumption which I had to prove, and this repeating process seemed to continue without end.

One morning I was walking on the street and waited for the traffic light to change.



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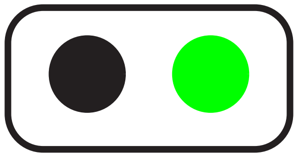


When I walked across on the road,
an example appeared in my mind automatically.

Counter Example (Toyama 1987)

$$R_1 \left\{ f(0,1,x) \to f(x,x,x) \right.$$

$$R_2 \left\{ \begin{array}{l} g(x,y) \to x \\ g(x,y) \to y \end{array} \right.$$

R_1 and R_2 are terminating but $R_1 \oplus R_2$ is not:

$$f(g(0,1),g(0,1),g(0,1)) \to$$

$$f(0,g(0,1),g(0,1)) \to$$

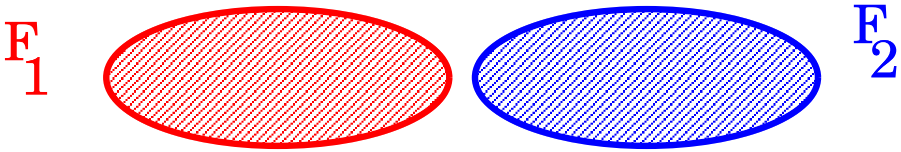
$$f(0,1,g(0,1)) \to$$

$$f(g(0,1),g(0,1),g(0,1)) \to \cdots$$

Thus termination is not modular.

Confluence Criteria for Non-Disjoint Union

R_1 and R_2 are confluent $\Longleftrightarrow$ $R_1 \oplus R_2$ is confluent.



Drawback:

The disjointness requirement $\mathcal{F}_1 \cap \mathcal{F}_2 = \phi$ is too strong.

Confluence Criteria for Non-Disjoint Union

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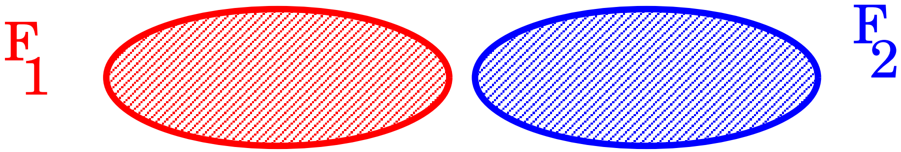
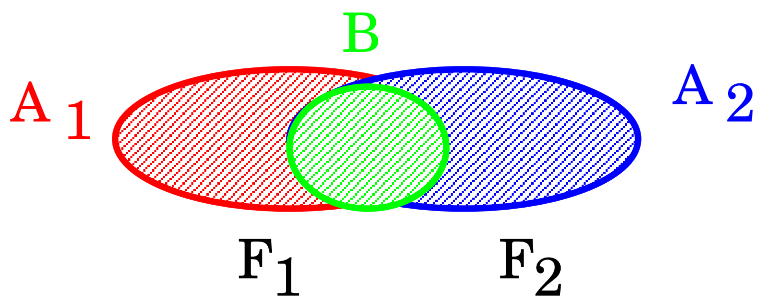


- Layer-preserving TRS (Ohlebusch 1994)
- Labeling (Zantema 1995, Toyama 1998)
- Persistence (Zantema 1994)
- Membership conditional TRS (Toyama 1988)
- Conditional linearization (Toyama and Oyamaguchi 1995)
- Non-E-overlapping TRS (Oyamaguchi and Ohta 1993)

Layer-Preserving TRS (Ohlebusch 1994)

R_1 and R_2 are layer-preserving and confluent

$\Rightarrow R_1 \cup R_2$ is confluent.



Let $B = F_1 \cap F_2$ and $A_i = F_i - B$ $(i = 1, 2)$.

$\mathcal{R}_i$ $(i = 1, 2)$ is layer-preserving if

(i) $\forall l \rightarrow r \in \mathcal{R}_i[root(l) \in A_i \Rightarrow root(r) \in A_i]$.

(ii) $\forall l \rightarrow r \in \mathcal{R}_i[root(l) \in B \Rightarrow l, r \in \mathcal{T}(B, V)]$.

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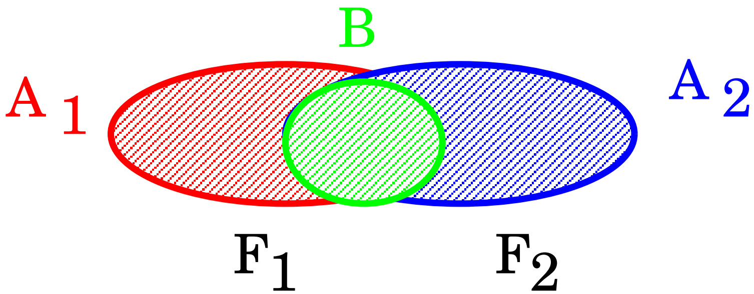

$$R \quad \begin{cases} f(x, a(g(x))) \longrightarrow g(f(x, x)) \\ f(x, g(x)) \longrightarrow g(f(x, x)) \\ a(x) \longrightarrow x \\ h(x) \longrightarrow h(a(h(x))) \end{cases}$$

is confluent since ...

Layer-Preserving TRS (Ohlebusch 1994)

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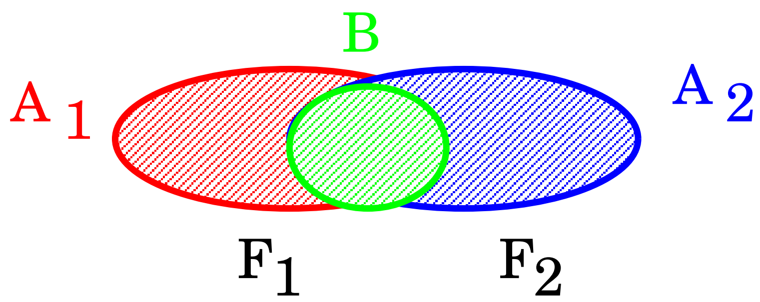


$$A_1 = \{f, g\},\ A_2 = \{h\}, B = \{a\}.$$

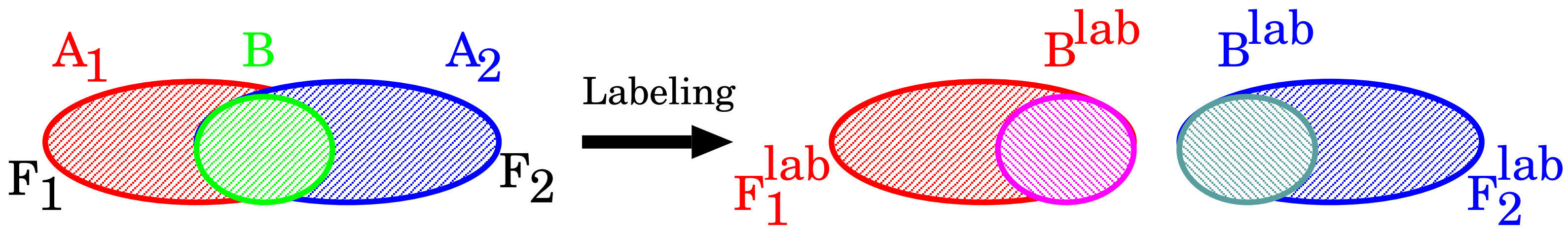
$$R_1 \quad \begin{cases} f(x, a(g(x))) \longrightarrow g(f(x, x)) \\ f(x, g(x)) \longrightarrow g(f(x, x)) \\ a(x) \longrightarrow x \end{cases}$$

$$R_2 \quad \{\ h(x) \longrightarrow h(a(h(x)))$$

are layer-preserving and confluent.

Top-Down Labeling (Toyama 1998)

R_1 and R_2 are layer-preserving $\Rightarrow R_1 \cup R_2 \simeq R_1^{\mathsf{lab}} \oplus R_2^{\mathsf{lab}}$.



$$R \quad \begin{cases} f(x, a(g(x))) \to g(f(x,x)) \\ f(x, g(x)) \to g(f(x,x)) \\ a(x) \to x \\ h(x) \to h(a(h(x))) \end{cases}$$

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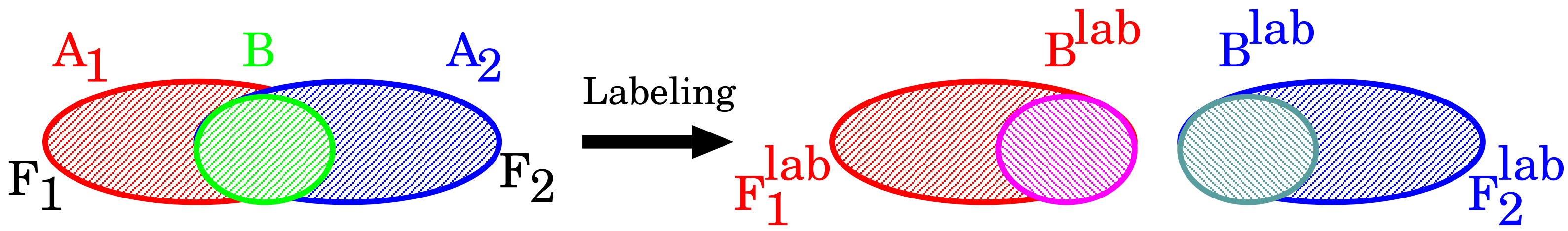


$$A_1 = \{f, g\},\ A_2 = \{h\}, B = \{a\}.$$

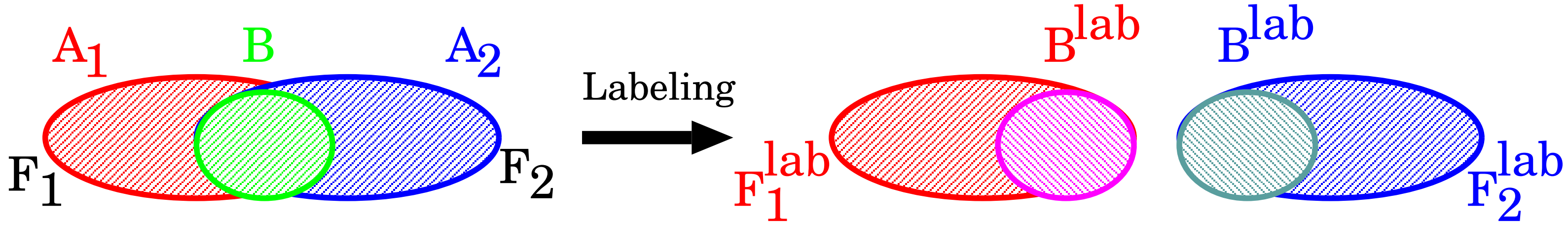
$$R_1^{\mathsf{lab}} \quad \begin{cases} f^1(x, a^1(g^1(x))) \to g^1(f^1(x,x)) \\ f^1(x, g^1(x)) \to g^1(f^1(x,x)) \\ a^1(x) \to x \end{cases}$$

$$R_2^{\mathsf{lab}} \quad \begin{cases} a^2(x) \to x \\ h^2(x) \to h^2(a^2(h^2(x))) \end{cases}$$

are disjoint and confluent.

Semantic Labeling (Zantema 1995)

R_1 and R_2 are compatible with labeling $\Rightarrow R_1 \cup R_2 \simeq R_1^{\mathsf{lab}} \oplus R_2^{\mathsf{lab}}$.



$$R \quad \begin{cases} a(f(x), y) \to f(a(f(y), x)) \\ a(b(x), y) \to a(x, b(x)) \\ a(g(x), x) \to g(b(g(x))) \end{cases}$$

is confluent since ...

Semantic Labeling (Zantema 1995)

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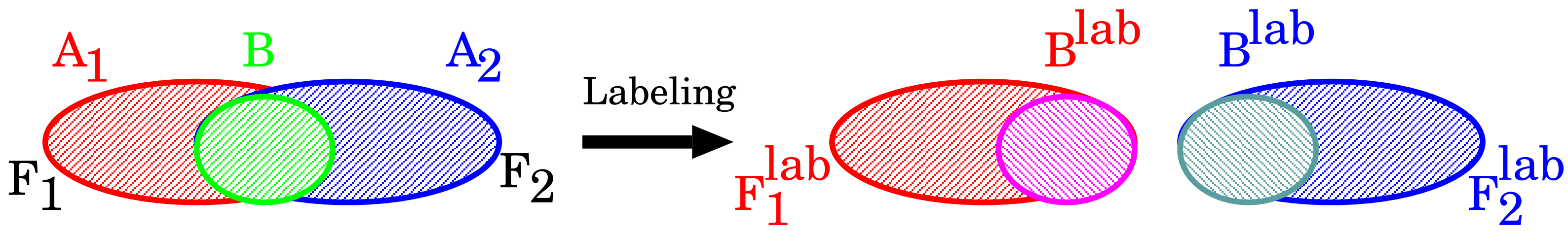


$$A_1 = \{f\},\ A_2 = \{g\}, B = \{a, b\}.$$

$$R_1^{\mathsf{lab}} \quad \begin{cases} a^1(f^1(x), y) \to f^1(a^1(f^1(y), x)) \\ a^1(b^1(x), y) \to a^1(x, b^1(x)) \end{cases}$$

$$R_2^{\mathsf{lab}} \quad \begin{cases} a^2(b^2(x), y) \to a^2(x, b^2(x)) \\ a^2(g^2(x), x) \to g^2(b^2(g^2(x))) \end{cases}$$

are disjoint and confluent.

Persistence (Zantema 1994)

R^τ is confluent for some typing $\tau \Rightarrow R$ is confluent.

(Aoto and Toyama 1997)

$$R \quad \begin{cases} f(x) \rightarrow g(x) \\ a(x,y) \rightarrow a(f(x), f(x)) \\ b(f(x), x) \rightarrow b(x, f(x)) \\ b(g(x), x) \rightarrow b(x, g(x)) \end{cases}$$

is confluent since R^τ is confluent for

$$\tau \quad \begin{cases} f : 1 \rightarrow 1 \\ g : 1 \rightarrow 1 \\ a : 1 \times 1 \rightarrow 2 \\ b : 1 \times 1 \rightarrow 3 \end{cases}$$

Membership Conditional Rewrite Rules

λ+$\{\delta x x \to \mathsf{T}\}$ is not confluent (Klop 1980), but

λ+δ is confluent (Church 1941)

$$\delta \begin{cases} \delta M M \to \mathsf{T} & \text{if } M \text{ is a closed normal form} \\ \delta M N \to \mathsf{F} & \text{if } M, N \text{ are closed normal forms and } M \not\equiv N \end{cases}$$

Membership Conditional TRS (Toyama 1988)

$$R \quad \begin{cases} f(x,x) \to 0 \\ f(g(x),x) \to 1 \\ 2 \to g(2) \end{cases}$$

is not confluent, but

$$R^{MC} \quad \begin{cases} f(x,x) \to 0 & \text{if } x \in T(F',V) \\ f(g(x),x) \to 1 & \text{if } x \in T(F',V) \\ 2 \to g(2) \end{cases}$$

is confluent, where $F' = \{f, g, 0, 1\}$.

Note that every term in $T(F',V)$ is closed and terminating w.r.t. reduction.

Membership Condition + Persistence

$$R \quad \begin{cases} f(x,x) \rightarrow f(g(x),x) \\ f(g(x),x) \rightarrow f(h(x),h(x)) \\ h(g(x)) \rightarrow g(g(h(x))) \end{cases}$$

is confluent, since:

$$R^{MC} \quad \begin{cases} f(x,x) \rightarrow f(g(x),x) & \text{if } x \in T(F',V) \\ f(g(x),x) \rightarrow f(h(x),h(x)) & \text{if } x \in T(F',V) \\ h(g(x)) \rightarrow g(g(h(x))) & \text{if } x \in T(F',V) \end{cases}$$

is confluent, where $F' = \{g,h\}$. Thus R^{τ} is confluent for

$$\tau \quad \begin{cases} f : 0 \times 0 \rightarrow 1 \\ g : 0 \rightarrow 0 \\ h : 0 \rightarrow 0 \end{cases}$$

Conditional Liniearization

R has unique normal form if R^L is confluent.
(de Vrijer and Klop 1989)

$R = CL + \{Dxx \to E\}$ has unique normal form since

conditional linearization
$R^L = CL + \{Dxx' \to E \text{ if } x = x'\}$ is confluent.

Conditional Liniearization

R has unique normal form if R^L is confluent.
(de Vrijer and Klop 1989)

A simple-right-linear R is confluent if R^L is non-overlapping.
(Toyama and Oyamaguchi 1995)

$$R \quad \begin{cases} f(x,x,y) \to h(y,c) \\ g(x) \to f(x,c,g(c)) \\ c \to h(c,c) \end{cases}$$

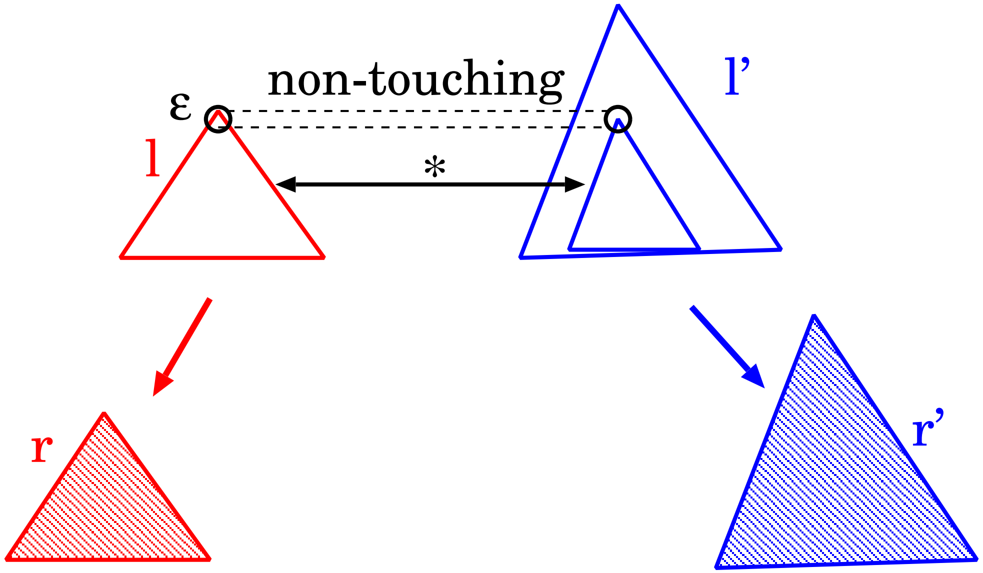
is confluent since

$$R^L \quad \begin{cases} f(x',x'',y') \to h(y,c) & \text{if } x'=x, x''=x, y'=y \\ g(x') \to f(x,c,g(c)) & \text{if } x'=x \\ c \to h(c,c) \end{cases}$$

is non-overlapping.

E-Overlapping and Strongly Overlapping

- We say that R is E-overlapping if



- We say that R is strongly overlapping if R^L is overlapping.

R is strongly overlapping if R is E-overlapping.
(Ogawa and Ono 1989)

Note that strongly overlapping is a decidable approximation of E-overlapping.

Confluence Criteria for Non-E-Overlapping TRS

Non-E-Overlapping Right-Ground TRS
(Oyamaguchi and Ohta 1993)

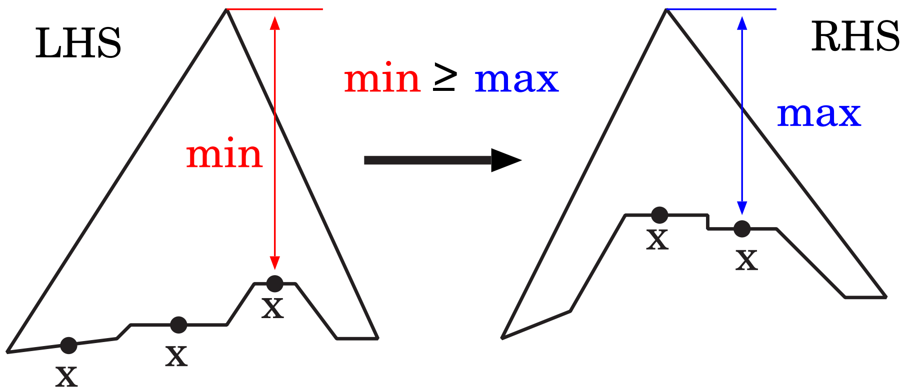
Non-E-Overlapping Simple-Right-Linear TRS
(Oyamaguchi and Toyama 1995)

Non-E-Overlapping Strongly Depth-Preserving TRS
(Gomi, Oyamaguchi, Ohta 1996)

Root-E-Closed Strongly Depth-Preserving TRS
(Gomi, Oyamaguchi, Ohta 1998)

Strongly Depth-Preserving TRS (Gomi et al 1996)

Non-E-overlapping strongly depth-preserving TRS is confluent.



$$R \quad \begin{cases} f(x,x) \to a \\ c \to h(c, g(c)) \\ f(g(x), g(x)) \to f(x, h(x, g(c))) \end{cases}$$

is confluent since

R is non-E-overlapping strongly depth-preserving.

Decidability of Confluence

Confluence is Decidable for:

Ground TRS (Dauchet et al. 1987, Oyamaguchi 1987)

Right-Ground TRS (Godoy, Tiwari, Verma 2004)

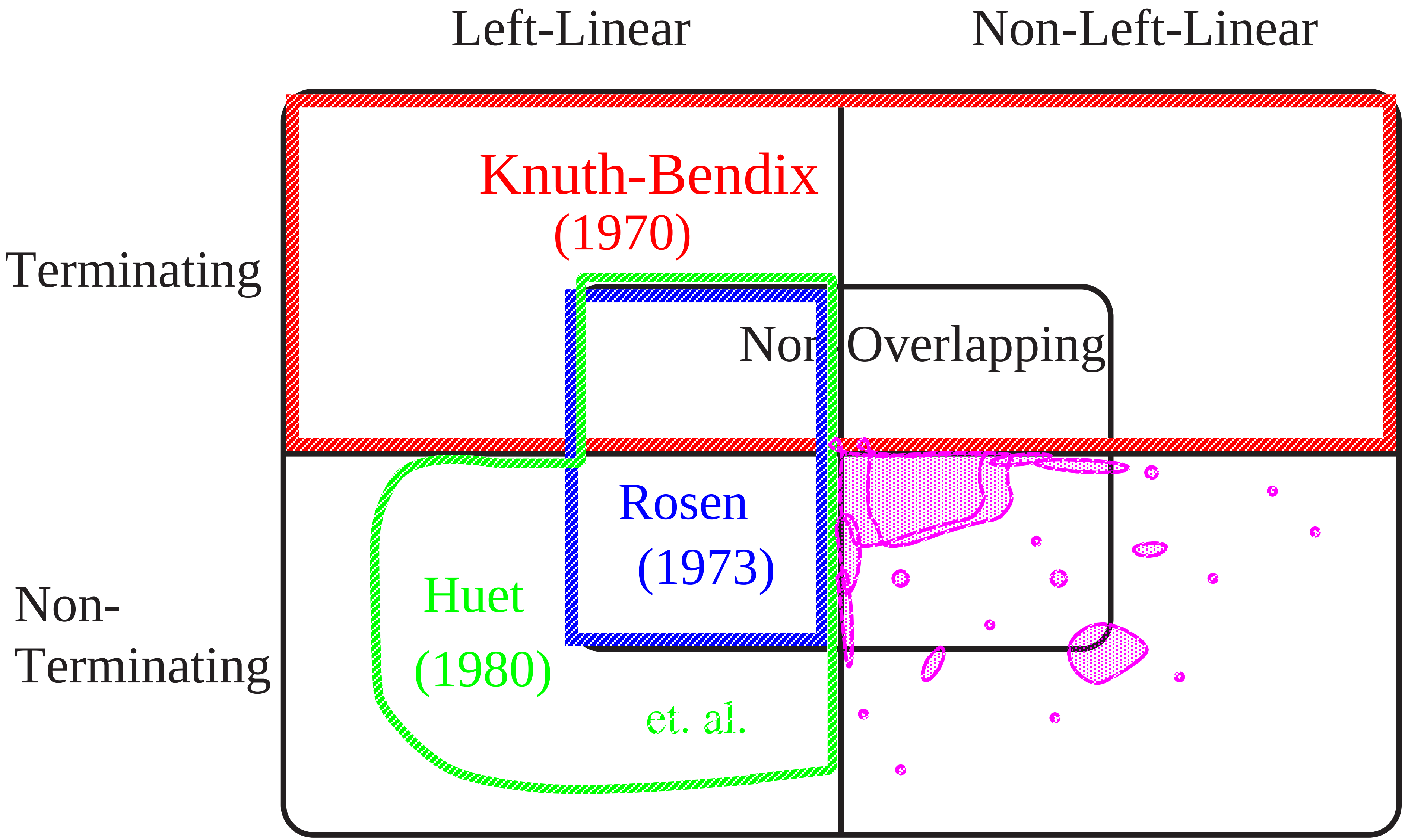
Right-(Ground or Variable) TRS (Godoy, Tiwari 2004)

Shallow Right-Linear TRS (Godoy, Tiwari 2005)

Confluence is Undecidable for:

Flat TRS (Jacquemard, 2003)

Confluence Criteria



Future Directions

For non-left-linear and non-terminating TRSs:

- New confluence criteria
- Relation among different proofs and techniques
- Theoretical characterization of confluence
- Automated provers for confluence

References

T. Aoto and Y. Toyama, Persistency of confluence, *Journal of Universal Computer Science, Vol. 3, No. 11* (1997) 1134-1147.

A. Church, *The calculi of lambda conversion* (Princeton University Press, 1941).

M. Dauchet, T. Heuillard, P. Lescanne, S. Tison, Decidability of the confluence of finite ground term rewrite systems and of other related term rewrite systems, *Proc. of LICS'87* (1987) 353-359.

K. Futatsugi and Y. Toyama, Term rewriting systems and their applications: A survey, *J. IPS Japan 24 (2)* (1983) 133-146, *in Japanese*.

G. Godoy, A. Tiwari, R. M. Verma, Characterizing confluence by rewrite closure and right ground term rewrite systems, *Appl. Algebra Eng. Commun. Comput. 15* (2004) 13-36.

G. Godoy, A. Tiwari, Deciding fundamental properties of right-(ground or variable) rewrite systems by rewrite closure, Proc. of IJCAR'04, Lecture Notes in Computer Science 3097 (2004) 91-106.

G. Godoy, A. Tiwari, Confluence of shallow right-linear rewrite systems, *Proc. of CSL'05, Lecture Notes in Computer Science 3634* (2005) 541-556.

H. Gomi, M. Oyamaguchi, Y. Ohta, On the Church-Rosser property of non-E-overlapping and depth-preserving TRS's, *Trans. of IPSJ, Vol.37, No.12* (1996) 2147-2160.

H. Gomi, M. Oyamaguchi, Y. Ohta, On the Church-Rosser property of root-E-overlapping and strongly depth-preserving term rewriting systems, *Trans. IPS. Japan, Vol.39, No.4* **(1998) 992-1005.**

B. Gramlich, Confluence without termination via parallel critical pairs, *Proc. of CAAP'96, Lecture Notes in Computer Science 1059* **(1996) 211-225.**

G. Huet, Confluent reductions: Abstract properties and applications to term rewriting systems, *J. ACM 27* **(1980) 797-821.**

F. Jacquemard, Reachability and confluence are undecidable for flat term rewriting systems, *Inf. Process. Lett. 87* **(2003) 265-270.**

J. W. Klop, Combinatory reduction systems, Dissertation, Univ. of Utrecht (1980).

D. E. Knuth and P. G. Bendix, Simple word problems in universal algebras, in: J. Leech, ed., *Computational problems in abstract algebra* **(Pergamon Press, 1970) 263-297.**

M. J. O'Donnell, Computing in systems described by equations, *Lecture Notes in Computer Science 58* **(1977).**

M. Ogawa and S. Ono, On the uniquely converging property of nonlinear term rewriting systems, *IEICE Technical Report, COMP89-7* **(1989) 61-70.**

E. Ohlebusch, Modular Properties of Composable Term Rewriting Systems, Dissertation, Univ. Bielefeld (1994).

S. Okui, Simultaneous critical pairs and Church-Rosser property, *Proc. of RTA'98, Lecture Notes in Computer Science 11379* (1998) 2-16.

V. van Oostrom, Development closed critical pairs, *Proc. of HOA '95, Lecture Notes in Computer Science 1074* (1995) 185-200.

M. Oyamaguchi, The Church-Rosser property for ground term-rewriting systems is decidable, *Theoretical Computer Science 49* (1987) 43-79.

M. Oyamaguchi and Y. Ohta, On the confluent property of right-ground term rewriting systems, *Trans. of IEICE, Vol.J76-D-I, No.2* (1993) 39-45, *in Japanese*.

M. Oyamaguchi and Y. Toyama, On the Church-Rosser property of E-overlapping and simple-right-linear TRS's, *Research Report of the Faculty of Engineering, Mie University, Vol. 20* (1995-12) 99-118.

M. Oyamaguchi and Y. Ohta, A new parallel closed condition for Church-Rosser of left-linear term rewriting systems, *Proc. of RTA'97, Lecture Notes in Computer Science 1232* (1997) 187-201.

M. Oyamaguchi and Y. Ohta, On the Church-Rosser property of left-linear term rewriting systems, *Trans. of IEICE, Vol.E86-D, No.1* (2003) 131-135.

B. K. Rosen, Tree-manipulating systems and Church-Rosser theorems, *J. ACM 20* (1973) 160-187.

J. Staples, Church-Rosser theorem for replacement systems, *Lecture Notes in Mathematics 450* (1975) 291-307.

Y. Toyama, On the Church-Rosser property of term rewriting systems, *NTT ECL Technical Report 17672* (1981-12), *in Japanese*.

Y. Toyama, On the Church-Rosser property for the direct sum of term rewriting systems, *Proc. of Lambda Calculus and Computer Science Theory Symposium (Kyoto, August 17-19, 1983), RIMS Kokyuroku 515* (1984) 110-133.

Y. Toyama, On the Church-Rosser property for the direct sum of term rewriting systems, *J. ACM 34* (1987) 128-143.

Y. Toyama, Counterexamples to termination for the direct sum of term rewriting systems, *Inform. Process. Lett. 25* (1987) 141-143.

Y. Toyama, Commutativity of term rewriting systems, in: K. Fuchi and L. Kott, eds., *Programming of Future Generation Computer II* (North-Holland, 1988) 393-407.

Y. Toyama, Confluent term rewriting systems with membership conditions, *Proc. of CTRS'88, Lecture Notes in Computer Science 308* (1988) 228-241.

Y. Toyama and M. Oyamaguchi, Church-Rosser property and unique normal form property of non-duplicating term rewriting systems, *Proc. of CTRS'95, Lecture Notes in Comput. Sci. 968* (1995) 316-331.

Y. Toyama, Analysis of term rewriting systems by labeling, *Proc. of LA symposium* (1998-07) 46-49, *in Japanese*.

H. Zantema, Termination of term rewriting: interpretation and type elimination, *J. of Symbolic Computation 17* (1994) 23-50.

H. Zantema, Termination of term rewriting by semantic labelling, *Fundamenta Informaticae 24* (1995) 89-105.

R. C. de Vrijer and J. W. Klop, Unique normal forms for lambda calculus with surjective pairing, *Information and Computation 80* (1989) 97-113.