2022年度 数理論理学 復習問題(11)

問題 1 述語論理式 A を $A = \forall \mathrm{y}\,(\mathtt{Q}(\mathtt{x},\mathtt{y}) \land \exists \mathtt{x}\,\mathtt{R}(\mathtt{x},\mathtt{y},\mathtt{z}))$ とおくとき，A における変数の自由出現と束縛出現をそれぞれ (わかるように印をつけて) 示せ.

問題 2 以下の述語論理式 A について，自由変数の集合 $\mathrm{FV}(A)$ を求めよ.

(1) $A = \forall \mathtt{x}\,\mathtt{P}(\mathtt{x},\mathtt{y}) \land \forall \mathtt{z}\,\mathtt{P}(\mathtt{y},\mathtt{z})$

(2) $A = \forall \mathtt{x}\,\mathtt{P}(\mathtt{x},\mathtt{y}) \land \forall \mathtt{y}\,\mathtt{P}(\mathtt{y},\mathtt{z})$

(3) $A = \forall \mathtt{x}\,(\mathtt{P}(\mathtt{x},\mathtt{y}) \land \forall \mathtt{y}\,\mathtt{P}(\mathtt{x},\mathtt{y}))$

問題 3 以下の述語論理式における束縛関係を図示するともに，自由出現を明示せよ.

(1) $\forall \mathtt{y}\,(\forall \mathtt{x}\,\mathtt{P}(\mathtt{x},\mathtt{y}) \land \exists \mathtt{x}\,\forall \mathtt{y}\,\mathtt{P}(\mathtt{y},\mathtt{x}))$

(2) $\forall \mathtt{y}\,(\exists \mathtt{x}\,\mathtt{P}(\mathtt{f}(\mathtt{x},\mathtt{y}),\mathtt{z}) \land \exists \mathtt{z}\,\mathtt{P}(\mathtt{x},\mathtt{f}(\mathtt{y},\mathtt{z})))$

(3) $\forall \mathtt{x}\,(\exists \mathtt{x}\,\mathtt{Q}(\mathtt{x}) \land \exists \mathtt{y}\,\mathtt{P}(\mathtt{x},\mathtt{y}) \land \mathtt{P}(\mathtt{x},\mathtt{y}))$

問題 4 以下の論理的同値性を同値変形を用いて示せ. (変形に用いた規則の名前も示すこと．)

(1) $\forall \mathtt{x}\,(\mathtt{P}(\mathtt{x}) \to \mathtt{Q}) \cong \exists \mathtt{x}\,\mathtt{P}(\mathtt{x}) \to \mathtt{Q}$

(2) $\exists \mathtt{x}\,\forall \mathtt{y}\,(\mathtt{P}(\mathtt{a},\mathtt{y}) \land \mathtt{P}(\mathtt{x},\mathtt{y})) \cong \forall \mathtt{y}\,\mathtt{P}(\mathtt{a},\mathtt{y}) \land \exists \mathtt{x}\,\forall \mathtt{y}\,\mathtt{P}(\mathtt{x},\mathtt{y})$

(3) $\neg\forall \mathtt{x}\,\mathtt{P}(\mathtt{x},\mathtt{y}) \land \forall \mathtt{y}\,\mathtt{P}(\mathtt{x},\mathtt{y}) \cong \exists \mathtt{u}\,\forall \mathtt{v}\,(\neg\mathtt{P}(\mathtt{u},\mathtt{y}) \land \mathtt{P}(\mathtt{x},\mathtt{v}))$

問題 5 次の述語論理式の冠頭標準形を求めよ.

$$\exists \mathtt{y}\,(\neg\forall \mathtt{x}\,\mathtt{P}(\mathtt{x},\mathtt{y}) \lor \forall \mathtt{x}\,\mathtt{P}(\mathtt{x},\mathtt{y}))$$

2022 年度 数理論理学 復習問題解答 (11)

問題 1 $\underline{\underline{\qquad}}$ 部が束縛出現，$\underline{\qquad}$ 部が自由出現：$\forall \mathtt{y}\,(\mathtt{Q}(\underline{\mathtt{x}}, \underline{\underline{\mathtt{y}}}) \land \exists \mathtt{x}\,\mathtt{R}(\underline{\underline{\mathtt{x}}}, \underline{\underline{\mathtt{y}}}, \underline{\mathtt{z}}))$.

問題 2

(1) $\mathrm{FV}(A) = \{\mathtt{y}\}$
(2) $\mathrm{FV}(A) = \{\mathtt{y}, \mathtt{z}\}$
(3) $\mathrm{FV}(A) = \{\mathtt{y}\}$

問題 3 束縛関係を矢印で，自由出現は下線 $\underline{\qquad}$ で示す．

(1)

$$\forall \mathtt{y}\,(\forall \mathtt{x}\,\mathtt{P}(\mathtt{x}, \mathtt{y}) \land \exists \mathtt{x}\,\forall \mathtt{y}\,\mathtt{P}(\mathtt{y}, \mathtt{x}))$$

(2)

$$\forall \mathtt{y}\,(\exists \mathtt{x}\,\mathtt{P}(\mathtt{f}(\mathtt{x}, \mathtt{y}), \underline{\mathtt{z}}) \land \exists \mathtt{z}\,\mathtt{P}(\underline{\mathtt{x}}, \mathtt{f}(\mathtt{y}, \mathtt{z})))$$

(3)

$$\forall \mathtt{x}\,(\exists \mathtt{x}\,\mathtt{Q}(\mathtt{x}) \land \exists \mathtt{y}\,\mathtt{P}(\mathtt{x}, \mathtt{y}) \land \mathtt{P}(\mathtt{x}, \underline{\mathtt{y}}))$$

問題 4

(1)

$$\begin{aligned}
\forall \mathtt{x}\,(\mathtt{P}(\mathtt{x}) \to \mathtt{Q}) &\cong \forall \mathtt{x}\,(\neg \mathtt{P}(\mathtt{x}) \lor \mathtt{Q}) && (\text{含意の法則})\\
&\cong \forall \mathtt{x}\,\neg \mathtt{P}(\mathtt{x}) \lor \mathtt{Q} && (\text{束縛の移動})\\
&\cong \neg \exists \mathtt{x}\,\mathtt{P}(\mathtt{x}) \lor \mathtt{Q} && (\text{ド・モルガンの法則})\\
&\cong \exists \mathtt{x}\,\mathtt{P}(\mathtt{x}) \to \mathtt{Q} && (\text{含意の法則})
\end{aligned}$$

(2)

$$\begin{aligned}
\exists \mathtt{x}\,\forall \mathtt{y}\,(\mathtt{P}(\mathtt{a}, \mathtt{y}) \land \mathtt{P}(\mathtt{x}, \mathtt{y})) &\cong \exists \mathtt{x}\,(\forall \mathtt{y}\,\mathtt{P}(\mathtt{a}, \mathtt{y}) \land \forall \mathtt{y}\,\mathtt{P}(\mathtt{x}, \mathtt{y})) && (\text{全称の分配})\\
&\cong \forall \mathtt{y}\,\mathtt{P}(\mathtt{a}, \mathtt{y}) \land \exists \mathtt{x}\,\forall \mathtt{y}\,\mathtt{P}(\mathtt{x}, \mathtt{y}) && (\text{束縛の移動})
\end{aligned}$$

(3)

$$\begin{aligned}
\neg \forall \mathtt{x}\,\mathtt{P}(\mathtt{x}, \mathtt{y}) \land \forall \mathtt{y}\,\mathtt{P}(\mathtt{x}, \mathtt{y}) &\cong \exists \mathtt{x}\,\neg \mathtt{P}(\mathtt{x}, \mathtt{y}) \land \forall \mathtt{y}\,\mathtt{P}(\mathtt{x}, \mathtt{y}) && (\text{ド・モルガンの法則})\\
&\cong \exists \mathtt{u}\,\neg \mathtt{P}(\mathtt{u}, \mathtt{y}) \land \forall \mathtt{y}\,\mathtt{P}(\mathtt{x}, \mathtt{y}) && (\text{束縛変数の名前替え})\\
&\cong \exists \mathtt{u}\,\neg \mathtt{P}(\mathtt{u}, \mathtt{y}) \land \forall \mathtt{v}\,\mathtt{P}(\mathtt{x}, \mathtt{v}) && (\text{束縛変数の名前替え})\\
&\cong \exists \mathtt{u}\,\forall \mathtt{v}\,(\neg \mathtt{P}(\mathtt{u}, \mathtt{y}) \land \mathtt{P}(\mathtt{x}, \mathtt{v})) && (\text{束縛の移動})
\end{aligned}$$

問題 5

$$\begin{aligned}
\exists \mathtt{y}\,(\neg \forall \mathtt{x}\,\mathtt{P}(\mathtt{x}, \mathtt{y}) \lor \forall \mathtt{x}\,\mathtt{P}(\mathtt{x}, \mathtt{y})) &\cong \exists \mathtt{y}\,(\exists \mathtt{x}\,\neg \mathtt{P}(\mathtt{x}, \mathtt{y}) \lor \forall \mathtt{x}\,\mathtt{P}(\mathtt{x}, \mathtt{y})) && (\text{ド・モルガンの法則})\\
&\cong \exists \mathtt{y}\,\exists \mathtt{x}\,(\neg \mathtt{P}(\mathtt{x}, \mathtt{y}) \lor \forall \mathtt{x}\,\mathtt{P}(\mathtt{x}, \mathtt{y})) && (\text{束縛の移動})\\
&\cong \exists \mathtt{y}\,\exists \mathtt{x}\,(\neg \mathtt{P}(\mathtt{x}, \mathtt{y}) \lor \forall \mathtt{z}\,\mathtt{P}(\mathtt{z}, \mathtt{y})) && (\text{束縛変数の名前替え})\\
&\cong \exists \mathtt{y}\,\exists \mathtt{x}\,\forall \mathtt{z}\,(\neg \mathtt{P}(\mathtt{x}, \mathtt{y}) \lor \mathtt{P}(\mathtt{z}, \mathtt{y})) && (\text{束縛の移動})
\end{aligned}$$