

2020 年度 数理論理学 復習問題 (6)

問題 1 以下の命題論理式の証明図を書け.

- (1) $(P \wedge Q) \vee P \rightarrow P$
- (2) $P \vee (Q \vee R) \rightarrow Q \vee (R \vee P)$
- (3) $P \vee (Q \wedge R) \rightarrow (P \vee Q) \wedge (P \vee R)$
- (4) $(P \vee Q \rightarrow R) \rightarrow (P \rightarrow R) \wedge (Q \rightarrow R)$
- (5) $(P \rightarrow Q) \vee (R \rightarrow S) \rightarrow P \wedge R \rightarrow Q \vee S$

問題 2 以下の命題論理式の証明図を書け.

- (1) $\perp \vee Q \rightarrow P \vee Q$
- (2) $(\neg Q \rightarrow \neg P) \rightarrow (\neg Q \rightarrow P) \rightarrow Q$
- (3) $P \vee (Q \rightarrow P) \rightarrow P \vee \neg Q$
- (4) $P \vee Q \leftrightarrow (\neg P \rightarrow Q)$
- (5) $\neg(P \wedge Q) \leftrightarrow \neg P \vee \neg Q$

2020年度 数理論理学 復習問題解答 (6)

問題 1

(1)

$$\frac{\frac{[(P \wedge Q) \vee P]^2 \quad \frac{[P \wedge Q]^1}{P} \wedge E \quad [P]^1}{P} \vee E^1}{(P \wedge Q) \vee P \rightarrow P} \rightarrow I^2$$

(2)

$$\frac{[P \vee (Q \vee R)]^3 \quad \frac{\frac{[P]^2}{R \vee P} \vee I \quad \frac{[Q \vee R]^2 \quad \frac{[Q]^1}{Q \vee (R \vee P)} \vee I \quad \frac{[R]^1}{R \vee P} \vee I}{Q \vee (R \vee P)} \vee E^1}{Q \vee (R \vee P)} \vee E^2}{P \vee (Q \vee R) \rightarrow Q \vee (R \vee P)} \rightarrow I^3$$

(3)

$$\frac{[P \vee (Q \wedge R)]^2 \quad \frac{\frac{[P]^1}{P \vee Q} \vee I \quad \frac{[P]^1}{P \vee R} \vee I}{(P \vee Q) \wedge (P \vee R)} \wedge I \quad \frac{\frac{[Q \wedge R]^1}{Q} \wedge E \quad \frac{[Q \wedge R]^1}{R} \wedge E}{\frac{Q}{P \vee Q} \vee I \quad \frac{R}{P \vee R} \vee I} \wedge I}{(P \vee Q) \wedge (P \vee R)} \vee E^1}{P \vee (Q \wedge R) \rightarrow (P \vee Q) \wedge (P \vee R)} \rightarrow I^2$$

(4)

$$\frac{\frac{[P \vee Q \rightarrow R]^3 \quad \frac{[P]^1}{P \vee Q} \vee I}{P \rightarrow R} \rightarrow E \quad \frac{[P \vee Q \rightarrow R]^3 \quad \frac{[Q]^2}{P \vee Q} \vee I}{Q \rightarrow R} \rightarrow E}{\frac{R}{P \rightarrow R} \rightarrow I^1 \quad \frac{R}{Q \rightarrow R} \rightarrow I^2} \wedge I}{(P \rightarrow R) \wedge (Q \rightarrow R)} \wedge I}{(P \vee Q \rightarrow R) \rightarrow (P \rightarrow R) \wedge (Q \rightarrow R)} \rightarrow I^3$$

(5)

$$\frac{[(P \rightarrow Q) \vee (R \rightarrow S)]^3 \quad \frac{\frac{[P \rightarrow Q]^1 \quad \frac{[P \wedge R]^2}{P} \wedge E}{Q} \rightarrow E \quad \frac{[R \rightarrow S]^1 \quad \frac{[P \wedge R]^2}{R} \wedge E}{S} \rightarrow E}{Q \vee S} \vee I \quad \frac{S}{Q \vee S} \vee I}{Q \vee S} \vee E}{P \wedge R \rightarrow Q \vee S} \rightarrow I^2}{(P \rightarrow Q) \vee (R \rightarrow S) \rightarrow P \wedge R \rightarrow Q \vee S} \rightarrow I^3$$

問題 2

(1)

$$\frac{[\perp \vee Q]^1 \quad \frac{[\perp]^2}{P \vee Q} \perp \quad \frac{[Q]^2}{P \vee Q} \vee I}{P \vee Q} \vee E^2}{\perp \vee Q \rightarrow P \vee Q} \rightarrow I^1$$

(2)

$$\begin{array}{c}
\frac{[\neg Q \rightarrow \neg P]^3 \quad [\neg Q]^1}{\neg P} \rightarrow E \quad \frac{[\neg Q \rightarrow P]^2 \quad [\neg Q]^1}{P} \rightarrow E \\
\frac{\perp}{Q} \text{RAA}^1 \\
\frac{(\neg Q \rightarrow P) \rightarrow Q}{(\neg Q \rightarrow \neg P) \rightarrow (\neg Q \rightarrow P) \rightarrow Q} \rightarrow I^2 \\
\rightarrow I^3
\end{array}$$

(3)

$$\begin{array}{c}
\frac{[P \vee (Q \rightarrow P)]^1 \quad \frac{[P]^2}{P \vee \neg Q} \vee I \quad \frac{Q \vee \neg Q}{P \vee \neg Q} \text{EM} \quad \frac{[Q \rightarrow P]^2 \quad [Q]^3}{P} \rightarrow E \quad \frac{[Q]^3}{P \vee \neg Q} \vee I}{\frac{P \vee \neg Q}{P \vee (Q \rightarrow P) \rightarrow P \vee \neg Q} \vee E^2} \vee I^3 \\
\rightarrow I^1
\end{array}$$

(4)

$$\begin{array}{c}
\frac{[\neg P]^1 \quad [P]^3}{\perp} \neg E \\
\frac{[P \vee Q]^4 \quad \frac{\perp}{Q} \perp}{\neg P \rightarrow Q} \rightarrow I^1 \quad \frac{[Q]^3}{\neg P \rightarrow Q} \rightarrow I^2 \\
\frac{\neg P \rightarrow Q}{P \vee Q \leftrightarrow (\neg P \rightarrow Q)} \vee E^3 \quad \frac{P \vee \neg P}{P \vee Q} \text{EM} \quad \frac{[P]^1}{P \vee Q} \vee I \quad \frac{[\neg P \rightarrow Q]^4 \quad [\neg P]^1}{Q} \rightarrow E \\
\frac{P \vee Q}{P \vee Q} \vee I^1 \quad \leftrightarrow I^4
\end{array}$$

(5)

$$\begin{array}{c}
\frac{P \vee \neg P}{Q \vee \neg Q} \text{EM} \quad \frac{[\neg(P \wedge Q)]^3 \quad \frac{[P]^2 \quad [Q]^1}{P \wedge Q} \wedge I}{\neg P \vee \neg Q} \perp \\
\frac{\neg P \vee \neg Q}{\neg P \vee \neg Q} \vee I^1 \quad \frac{[\neg Q]^1}{\neg P \vee \neg Q} \vee I^1 \quad \frac{[P]^2}{\neg P \vee \neg Q} \vee I^2 \\
\frac{[\neg P \vee \neg Q]^3 \quad \frac{[P \wedge Q]^2}{P} \wedge E \quad \frac{[Q]^1}{Q} \wedge E}{\perp} \neg E \\
\frac{\perp}{\neg(P \wedge Q)} \neg I^2 \quad \leftrightarrow I^3 \\
\neg(P \wedge Q) \leftrightarrow \neg P \vee \neg Q
\end{array}$$