

2024 年度 数理論理学 復習問題 (7)

問題 1 以下の命題論理式の証明図を示し，対応する擬似証明コードを書け．

(1) $P \rightarrow (P \rightarrow Q) \rightarrow Q$

(2) $(Q \rightarrow R) \rightarrow Q \wedge P \rightarrow R$

(3) $(Q \rightarrow Q \rightarrow \perp) \rightarrow \neg Q$

(4) $(Q \rightarrow P) \rightarrow (P \vee Q) \rightarrow P$

(5) $(P \rightarrow Q) \rightarrow (P \wedge Q \leftrightarrow P)$

2024 年度 数理論理学 復習問題解答 (7)

問題 1

(1)

証明図:

$$\frac{\frac{\frac{[P \rightarrow Q]^1 \quad [P]^2}{Q} \rightarrow E}{(P \rightarrow Q) \rightarrow Q} \rightarrow I^1}{P \rightarrow (P \rightarrow Q) \rightarrow Q} \rightarrow I^2$$

擬似証明コード (解答例):

```
assume 1 : P → Q and 2 : P
hence Q by →E
hence P → (P → Q) → Q by →I[1, 2]
```

(2)

証明図:

$$\frac{\frac{\frac{[Q \rightarrow R]^2 \quad \frac{[Q \wedge P]^1}{Q} \wedge E}{R} \rightarrow E}{Q \wedge P \rightarrow R} \rightarrow I^1}{(Q \rightarrow R) \rightarrow Q \wedge P \rightarrow R} \rightarrow I^2$$

擬似証明コード (解答例):

```
assume 1 : Q → R and 2 : Q ∧ P
from 2 have Q by ∧E
hence R using 1 by →E
hence (Q → R) → Q ∧ P → R by →I[2, 1]
```

(3)

証明図:

$$\frac{\frac{\frac{[Q \rightarrow Q \rightarrow \perp]^2 \quad [Q]^1}{Q \rightarrow \perp} \rightarrow E}{\frac{\perp}{\neg Q} \neg I^1} \rightarrow E}{(Q \rightarrow Q \rightarrow \perp) \rightarrow \neg Q} \rightarrow I^2$$

擬似証明コード (解答例):

```
assume 1 : Q → Q → ⊥ and 2 : Q
hence ⊥ by →E
hence ¬Q by ¬I[2]
hence (Q → Q → ⊥) → ¬Q by →I[1]
```

(4)

証明図:

$$\frac{\frac{\frac{[P \vee Q]^2 \quad [P]^1}{P} \rightarrow I^2 \quad \frac{\frac{[Q \rightarrow P]^3 \quad [Q]^1}{P} \rightarrow E}{P} \vee E^1}{(P \vee Q) \rightarrow P} \rightarrow I^2}{(Q \rightarrow P) \rightarrow (P \vee Q) \rightarrow P} \rightarrow I^3$$

擬似証明コード (解答例):

```
assume 1 : P ∨ Q and 2 : Q → P
show P as follows:
  distinguish cases using 1
  case P
    hence P trivially
  case Q
    hence P using 2 by →E
  hence claim by ∨E
hence (Q → P) → (P ∨ Q) → P by →I[1, 2]
```

(5)

証明図:

$$\frac{\frac{\frac{[P \wedge Q]^1}{P} \wedge E \quad \frac{\frac{[P]^1 \quad \frac{[P \rightarrow Q]^2 \quad [P]^1}{Q} \rightarrow E}{P \wedge Q} \wedge I}{P \wedge Q \leftrightarrow P} \leftrightarrow I^1}{(P \rightarrow Q) \rightarrow (P \wedge Q \leftrightarrow P)} \rightarrow I^2$$

擬似証明コード (解答例):

```
assume 1 : P → Q
show P ∧ Q ↔ P as follows:
  (=⇒)
    assume 2 : P ∧ Q
    hence P by ∧E
  (⇐=)
    assume 3 : P
    hence Q using 1 by →E
    hence P ∧ Q using 3 by ∧I
  hence claim by ↔I[2, 3]
hence (P → Q) → (P ∧ Q ↔ P) by →I[1]
```